ST PAUL'S SCHOOL
JUNIOR SCHOLARSHIP EXAMINATION

MAY 2013

MATHEMATICS

2 hours

Answer as many questions as you can in any order you wish.

Question 10 should be answered on the insert sheet, and this should be handed in with the rest of your answers.

Credit will be given for reasoning and working where appropriate.

Give formulae for areas and volumes. When the answer is a fraction it should be given in mixed form, e.g. $3\frac{4}{5}$

The total number of marks for this paper is 109.
The mark allocation is shown in brackets at the end of each part of each question.

Please write your name on top of your answer sheets.

CALCULATORS MAY NOT BE USED.

1 Work out the values of:

(i) $2+3\times5$ [1]

(ii) $0.3 - 0.2 \div 8$, giving your answer as a fraction in its lowest terms, [2]

(iii) $5\frac{1}{4} \div 1\frac{1}{6}$, giving your answer as a mixed fraction in its lowest terms, [2]

(iv) $\left(\frac{1}{6}+\frac{1}{7}\right)\times1\frac{31}{39}$ giving your answer as a fraction in its lowest terms. [2]

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2 The following pairs of expressions are equivalent – that is, they are identical for all values of x. For example, if $mx + 5 = 3x + n$ for all x, then $m = 3$ and $n = 5$. Find the values of a, b, c, d and e.

(i) $\frac{5}{7}(x+3) = \frac{5(x+3)}{a}$

(ii) $\frac{3x}{4} = \frac{12x}{b}$

(iii) $(2x - 3)(5x + 2) = cx^2 + dx + e$ [5]

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3 Find the values of the following expressions when $x = -2$, $y = -1$, $z = -8$.

(i) $2x - z$ [1]

(ii) $x^2 - z^2$ [1]

(iii) $\frac{x}{yz}$ [1]

(iv) y^{xz} [1]

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4 Solve the following equations for x, giving answers as exact fractions where appropriate:

(i) $3+\frac{4}{x}=15$ [2]

(ii) $\frac{x+3}{2x-5}=2$ [2]

(iii) $(x+2)^2 = x^2 + 2$ [3]

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5 (a) Ashwin travels for 3 hours at 25 miles per hour (mph) and for 60 miles at 40 mph. Find his average speed. [2]

(b) Fred travels for 2 hours at 40 mph and 3 hours at x mph. Fred's average speed is 34 mph. Find the value of x. [3]

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6 (a) There are 10 boys, 12 girls and a teacher in a classroom. The boys have an average (mean) of 3 pens each. The girls have an average of 7 pens each and the teacher has m pens. The average number of pens that each person in the room has is 6. Find the value of m. [3]

(b) For the set of numbers

$$8, -3, 3, 6, 5, n$$

where n is a whole number, find all the possible values of the median of the set. [3]

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7 We define the operation * by $a*b = 3a+b-2$.

(i) Find the value of $(2 * 3) * 4$. [1]

(ii) Find the value of $2 * (3 * 4)$. [1]

(iii) Solve the equation $(x+2)*5 = 18$. [2]

(iv) If $x*y = y*x$ what can be said about x and y ? [2]

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8 The numbers x and y are written as products of prime numbers as follows:

$$x = 2^7 \times 3^5 \times 5^2, \quad y = 2^2 \times 3^4 \times 7^5.$$

The highest common factor of x and y is denoted by HCF(x, y), and the lowest common multiple of x and y is denoted by LCM(x, y).

(i) Find HCF(x, y), leaving your answer as a product of powers of prime numbers. [1]

(ii) Find LCM(x, y), leaving your answer as a product of powers of prime numbers. [1]

(iii) Show that, in this case, HCF(x, y) × LCM(x, y) = xy. [2]

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9 (i) Work out 13×17. [1]

(ii) You are given that

$$4n^4 + 1 = (2n^2 + 2n + 1)(2n^2 - 2n + 1).$$

Use this result to write 40001 as a product of prime factors. [4]

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10 ***Answer this question on the insert sheet provided.***

TURN OVER

11 At Pierre's patisserie, five pains-au-chocolat and two plain croissants cost £12. Three plain croissants and two pains-au-chocolat cost £8.10. Two almond croissants cost 70p more than a plain croissant and a pain-au-chocolat. Find the cost of each kind of pastry. [5]

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12 I throw two fair six-sided dice. I add up the two numbers to obtain my score.

(i) What is the probability that my score is seven? [2]

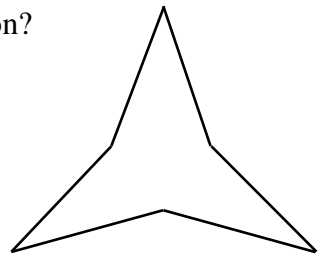
(ii) What is the probability that my score is a prime number? [2]

(iii) What is the probability that at least one of the two numbers is a six? [2]

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13 (i) What is the sum of the interior angles of a hexagon? [1]

A hexagon with three acute interior angles is shown.



By using your answer to part (i), where appropriate, answer the following.

(ii) Can a hexagon have four acute interior angles? Justify your answer. [2]

(iii) Can a hexagon have five acute interior angles? Justify your answer. [2]

(iv) Can a heptagon have six acute interior angles? Justify your answer. [3]

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14 Each year I buy each of my nieces a present for Christmas. In 2010 I had five nieces and each present had a retail value of £8.00 which excluded 17.5% VAT.

(i) How much VAT did I pay in 2010? [2]

Three things happened during 2011 which affected my Christmas shopping bill.

- The pre-tax value of the gifts I wanted to buy was 30% higher than in 2010.
- VAT rose from 17.5% to 20%
- My sister gave birth to twin girls.

(ii) What was the pre-tax value of each gift I bought in 2011? [2]

(iii) How much did I spend in total on gifts for my nieces in 2011 (including tax)? [3]

(iv) What was the percentage increase in the amount of VAT I paid in 2011 compared with 2010? [4]

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15 (a) A rectangular piece of paper $ABCD$ has dimensions $AB = 12$ cm and $BC = 5$ cm (see Fig 1). It is rolled into the shape of a vertical cylinder, with A joined to B and C joined to D. AB rests on a horizontal table. An ant starts at A and crawls upwards round the cylinder in a spiral. His angle to the horizontal never changes, and when he reaches the top he is at D, directly above his starting point for the first time (see Fig. 2). Find the distance that the ant travels. [4]

Fig. 1

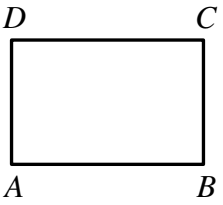
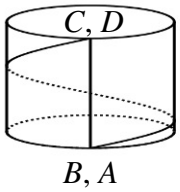


Fig. 2



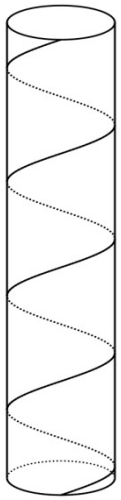
(b) Fig. 3 shows a cylindrical pole with radius 1 cm and height 24π cm. Another ant decides to crawl up and down the pole.

On the way up, she crawls in a spiral which goes 9 times round the pole (Fig. 3 shows only 4 times round) and finishes above her starting point.

On the way down, she crawls in a spiral which goes n times round the pole and finishes back where she started.

On each stage of the journey her angle to the horizontal never changes, but of course that angle is different on the way down.

Fig. 3



(i) Find the exact distance she travels up the pole. [4]

(ii) The distance she travels down is $\frac{4}{3}$ times the distance she travels up. Find the value of n. [4]

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16 A sequence $u_1, u_2, u_3, \ldots$ has nth term $u_n = 4n + (-1)^n$.

So, for example, $u_1 = 4\times1 + (-1)^1 = 3$, and $u_2 = 4\times2 + (-1)^2 = 9$.

(i) Find the values of u_3, u_4, u_5. [3]

(ii) Find a formula for the nth term of each of the following sequences

(a) 100, 93, 86, 79, 72, 65, … [2]

(b) 7, 11, 19, 23, 31, 35, … [3]

(c) 0, 5, 8, 17, 24, 37, 48, 65, … [3]

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